

# Reply to Boglione and Pennington

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## Abstract

In a recent article we presented an argument which, we believe, shows to be incorrect an estimate, by Boglione and Pennington, of corrections to the valence (quenched) approximation predictions for properties of the lightest scalar glueball. Boglione and Pennington's reply to our article, it appears to us, fails to address the specific technical issues we raised.

In Ref. [1] a formula is assumed for the errors, in valence (quenched) approximation scalar glueball predictions, which arise from the valence approximation's omission of glueball-quarkonium mixing. No derivation from QCD itself is given to suggest why this particular formula for valence approximation errors might be expected to hold. The errors are then evaluated in a model. In Ref. [2] we raised two main objections to the work of Ref. [1] and, as a consequence of these objections, concluded that the claims of Ref. [1] to have shown the valence approximation to be unreliable for properties of the lightest scalar glueball are not correct. Boglione and Pennington's [3] reply to our article mentions only one of our two objections and, it appears to us, fails to address the key technical issue on which this objection rests. To the second problem discussed in Ref. [2], Boglione and Pennington's reply offers no response.

In the present comment we will give a brief qualitative summary of the issues raised in Ref. [2], reply to Boglione and Pennington's [3] response to one of these issues, and, finally, consider an additional question concerning the work of Ref. [1] which arises from the numerical lattice calculation of glueball-quarkonium mixing in Ref. [4].

One of the objections to Ref. [1] discussed in Ref. [2] is a by-product of a systematic expansion derived in Ref. [2] for full QCD vacuum expectation values. The first term in this expansion is the valence approximation. Higher terms then give the valence approximation's error. Comparing these terms with the formula assumed for valence approximation errors in Ref. [1], we show in Ref. [2] that the error formula of Ref. [1] appears not to be a correct expression for the error in the valence approximation as it is generally applied.

The difficulty with the error formula of Ref. [1] has a simple origin. In lattice calculations for the valence approximation to QCD and for full QCD, with a common choice of lattice spacing and a common choice of renormalization conditions, the valence approximation QCD coupling constant  $g_{val}$  and the full QCD coupling constant  $g$  are not equal. The valence approximation  $g_{val}$  may be thought of as  $g$  divided by a chromoelectric analog of a dielectric constant taking into account the screening of  $g$  by dynamical quark-antiquark pairs present in full QCD but absent in the valence approximation. In the expansion for full QCD of Ref. [2], the difference between  $g$  of full QCD and  $g_{val}$  of the expansion's first term is compensated by counter-terms subtracted from the expansion's higher terms which contain closed quark loops. By choosing  $g_{val}$  entering the leading term so that full QCD and the valence approximation obey the same renormalization conditions, the valence approximation's accuracy is maximized and, by means of the subtracted counter-terms, the correction terms are minimized. Phrased differently, an optimally chosen  $g_{val}$  shifts as large a contribution as possible out of the expansion's higher order correction terms and into the expansion's first term.

In fact, however, the expansion remains logically correct for any choice of  $g_{val}$ . In particular,  $g_{val}$  could be set equal to the full QCD  $g$ . With this choice of  $g_{val}$ , the valence approximation becomes less accurate and the expansion's correction terms become larger. In particular, the counter-terms subtracted from quark loops in the error terms of the expansion entirely vanish. In the relation assumed in Ref. [1] for valence approximation errors, no counter-terms accompany quark loops in the error terms. As a consequence of this absence, the version of the valence approximation to which this formula applies must have  $g_{val}$  equal to  $g$ . The resulting valence approximation will be unnecessarily inaccurate and will differ from the valence approximation as generally applied in lattice QCD. Correspondingly, the

accompanying error terms will be unnecessarily large and will not, as claimed in Ref. [1], be correct estimates of the errors in the valence approximation with an optimal choice of  $g_{val}$ . For example using data of Ref. [5] for inverse lattice spacing of about 1 GeV, with  $g_{val}$  forced to  $g$ , light hadron masses and meson decay constants differ from experiment by as much as 45%, rather than by less than 10% or less than 20%, respectively, with an optimal choice of  $g_{val}$ .

Further evidence mentioned in Ref. [2] that the approximation to QCD which Ref. [1] identifies as the valence approximation uses a poorly chosen  $g_{val}$  is provided by the sign predicted in Ref. [1] for the error in valence approximation mass predictions. The valence approximation with correctly chosen  $g_{val}$ , as discussed in Ref. [6], will generally underestimate excited state masses and meson decay constants. This expectation is supported by the data of Refs. [5,7]. On the other hand, consider full QCD expanded according to Ref. [2] but with the valence approximation first term using full QCD  $g$  in place of the optimal screened  $g_{val}$ . As expected for the effect of quark-antiquark color screening and easily confirmed by an inspection of lattice data,  $g_{val}$  is less than  $g$ . Thus the mass predictions of this first term will be the valence approximation's predictions if, with no change in the value of lattice spacing,  $g_{val}$  is pushed up to  $g$ . A simple argument based on asymptotic freedom or another direct inspection of lattice data show that raising  $g_{val}$  to  $g$  increases mass predictions. Thus while the valence approximation with optimal  $g_{val}$  underestimates excited state masses, the valence approximation with  $g$  in place of  $g_{val}$  overestimates masses. This second alternative is the error found in the mass predictions of the proposed valence approximation of Ref. [1].

Boglionne and Pennington [3] offer, in effect, a two part response to the preceding discussion. First, they argue that there is a complex relation between their expansion, relying on an effective theory of interacting hadrons, and QCD itself, as an interaction of quarks and gluons. As a consequence, formulating the counter-terms of Ref. [2] in the language of Ref. [1] and identifying where they might occur in the arithmetic of Ref. [1] is difficult. Second, they argue that, by the nature of its construction as an effective field theory, their theory will somewhere include the counter-terms of Ref. [2] in so far as these are actually components of QCD. In reply to the first of these comments, however, we offer the observation of Ref. [2] that the counter-term to the one-loop quark diagram is simply a derivative of the leading valence approximation term with respect to  $g_{val}$ . Thus by expressing the parameters of the effective theory as functions of  $g_{val}$  it should, in principle, be possible to express the one-loop counter-term in an effective field theory even though quark and gluon fields themselves are submerged. In reply to the second comment, meanwhile, we point out that the question is not whether the counter-terms are implicitly present but rather exactly where they are present. With a correctly chosen  $g_{val}$  the counter-terms appear subtracted from valence approximation corrections and shift part of the would-be corrections forward into the valence approximation. With a  $g_{val}$  set equal to  $g$ , however, the counter-terms are missing from the quark-loop corrections and have been hidden, implicitly, in the valence approximation itself. The valence approximation then becomes less accurate and the corrections become larger. It is this second state of affairs which we believe we have shown occurs in the work of Ref. [1]. Whatever errors Ref. [1] may find in the valence approximation as formulated by them do not bear on the valence approximation as generally applied.

The other objection raised in Ref. [2] can be recast as the claim that the effective field theory on which the calculation of Ref. [1] is based, according to Boglionne and Pennington's

reply, is missing an important set of terms coupling glueballs and quarkonium. Boggione and Pennington's reply proposes that the lightest scalar quarkonium and glueball fields entering the Lagrangian of their effective field theory correspond to orthogonal states, respectively a pure, ideally mixed quarkonium nonet and a pure unmixed glueball. Mixing among the glueball and quarkonium states then occurs only through decay couplings linear in the scalar fields and second order in pseudoscalar quarkonium fields. Missing from this picture, however, are bilinear terms in the effective Lagrangian directly coupling the scalar glueball field to scalar quarkonium fields. Such terms are the effective field theory's remnants of the QCD quark-antiquark annihilation process for which a typical Feynman diagram is shown in Figure 1. The coefficients of these terms can not be determined from an effective field theory and must be taken as additional inputs from some other source. These terms are calculated using lattice QCD in Ref. [4] and shown to make significant contributions to quarkonium-glueball mixing. The mixing arising from these terms explains several otherwise puzzling features of observed scalar glueball and quarkonium data. It is shown further in Ref. [2] that mixing through these terms is quite probably more important than mixing through pseudoscalar pairs considered in Ref. [1] if mixing through pairs is calculated according to the systematic expansion of Ref. [2]. To the objection that Ref. [1] omits the dominant mixing process, Boggione and Pennington's reply offers no response.

A third issue concerning Ref. [1] arises from the calculation in Ref. [4] of the consequences of the mixing process of Fig. 1 for the glueball decay couplings calculated in Ref. [8] and taken as input by Ref. [1]. It is explained in Ref. [4] that the coupling calculation of Ref. [8] includes the first order effect of quarkonium-glueball mixing by the process of Fig. 1. If this first order effect is removed, Ref. [4] shows, it appears possible that the remaining coupling of the pure pseudoscalar glueball to pseudoscalar pairs may be zero. In this case, glueball-quarkonium mixing by the mechanism considered in Ref. [1] would not only be small in comparison to the contribution of Fig. 1, it would be zero.

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FIGURES

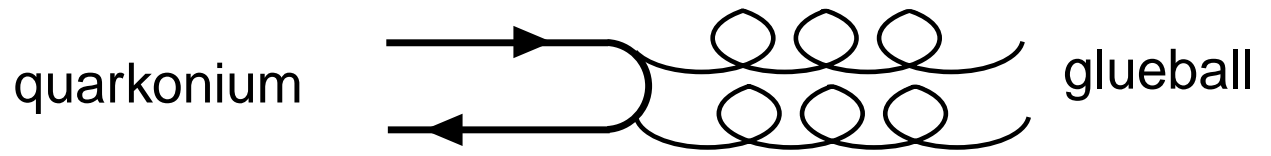


FIG. 1. Quarkonium-glueball mixing through quark-antiquark annihilation.